Multivariate: Factor analysis

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1 Goals

1.1 Goals

1.1.1 Goals of this lecture

- Factor analysis (FA)
  - **Dimension reduction**: reduce number of variables
- A **large set of (potentially correlated) observed variables**
  - Organize the **covariance** in those variables to a **smaller set** of orthogonal (uncorrelated) variables
- Similar to PCA but
  - **Assumptions** of FA are closer to what we expect in psychology

2 PCA vs FA

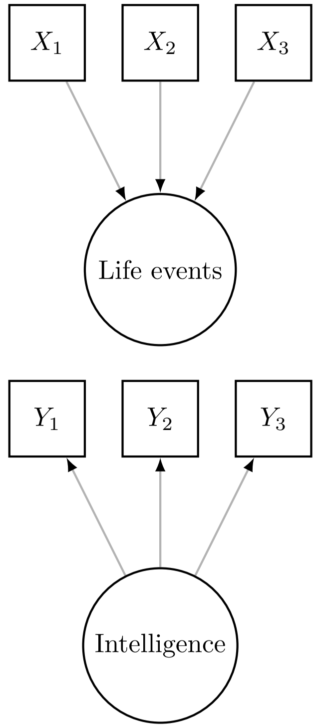
2.1 Statistical measurement

2.1.1 Measuring things is hard

- Psychology: we **cannot directly measure** some constructs
  - No ruler to measure "intelligence" or "introversion"
- We can **indirectly** measure what we really want to measure
  - Want to measure **intelligence**
    - Math ability, verbal ability, spatial ability, reasoning, general knowledge, etc.
  - Intelligence is a **latent variable**
    - Not *directly* observed

2.1.2 Formative vs reflective latent variables

- Formative factor
- Reflective factor



2.1.3 Measurement theory

- Psychometric theory: Latent variable is "true score"
- Observed score (Y) is a function of **true score** and **error**
  - True score is "real" score assuming no error (latent variable)
  - Error can be measurement error or random error

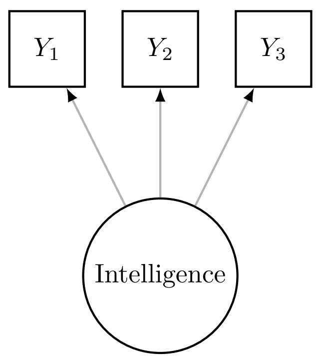
2.1.4 Measurement theory

- Observed score: $Y_i = T_i + e_{ij}$
  - True score (latent variable): T_i
  - Error: e_{ij}
- Variance of score:
  - $Var(Y_i) = Var(T_i) + Var(e_{ij})$
  - (Assuming no covariance between true score and error, which we do assume)

2.1.5 Measurement theory

- $Y_i = T_i + e_{ij}$
- FA partitions variance in each item into
  - **Common portion**: due to latent factors / true scores
  - **Unique portion**: due to error
- Big idea in **factor analysis**: Any correlations between items are due to what they have in common (i.e. a **common latent factor**)

2.1.6 Measurement theory



2.2 Differences between PCA and FA

2.2.1 Similar models, important differences

- Both PCA and FA are **data reduction** methods
  - You have many variables
  - You want to describe them using fewer dimensions
- Both PCA and FA are also **latent variable** methods
  - You have observed variables
  - They're related to unobserved (latent) variables
- Beyond that, several important **theoretical** and **statistical** differences

2.2.2 Common and unique variance

- PCA: **All variance is explained by latent variables**
  - If you retained all components, you'd *perfectly* re-create the observed variables
  - *Initial* communalities = 1.00
- FA: **Variance is divided into common and unique (error)**
  - Even if you retained all factors, there's still **measurement error**
  - Latent variables *never* explain *all* variance in observed
  - *Initial* communalities < 1.00

2.2.3 Causal ordering

- PCA: **Observed variables cause latent variables**
  - Latent variable is linear combination of observed
- FA: **Latent variables cause responses on observed variables**
  - Latent variable is a trait that causes a person to respond to the observed variables in a certain way

2.2.4 Variance or covariance / correlation?

- PCA: **Partitions variance in each variable**
  - Correlation / covariance largely ignored
  - **Variables don't even need to be correlated**
- FA: **Partitions variance, but in the service of splitting into common and unique portion**
  - Correlations between variables define "common variance"
  - **Variables related to the same factor are correlated**

2.3 Summary

2.3.1 Summary

- Factor analysis and PCA are similar
  - FA assumes measurement error
  - FA assumes latent factors cause responses
    - FA relies on correlations to get at this

3 Data Example

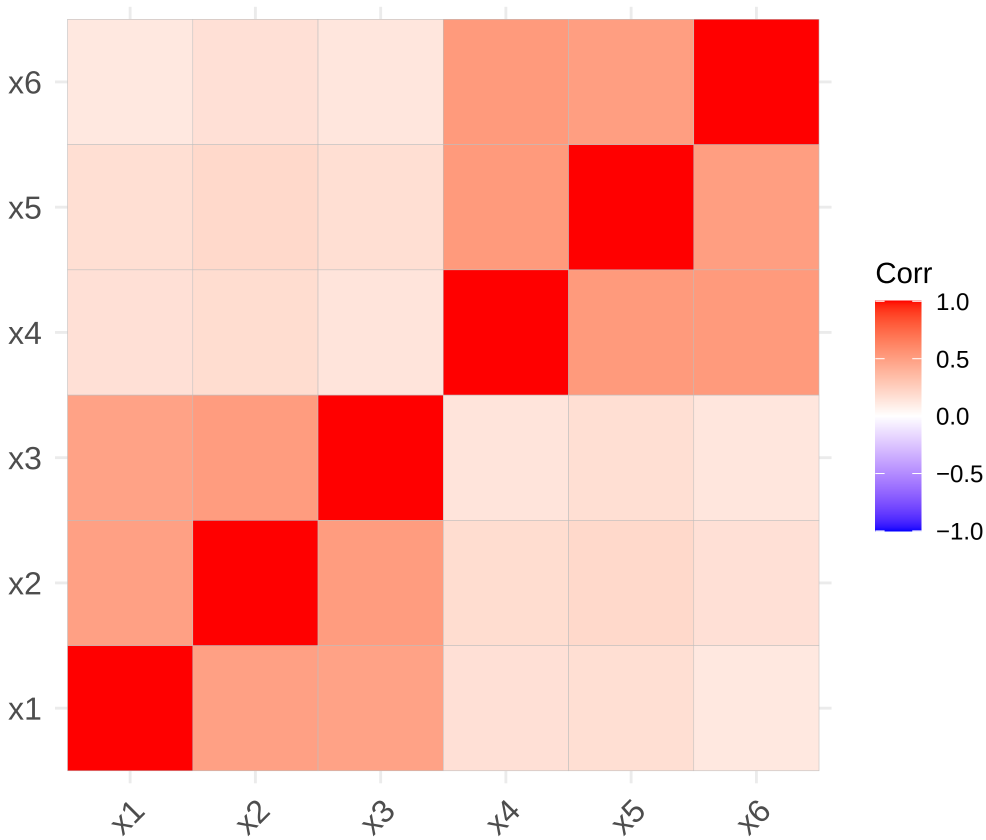
3.1 Measure and variables

3.1.1 Simulated data

- Similar to previous data
  - **1000 subjects**
  - 6 continuous variables
- Covariance matrix

```
      x1    x2    x3    x4    x5    x6
x1 1.871 0.912 0.944 0.312 0.344 0.226
x2 0.912 1.830 0.994 0.367 0.385 0.315
x3 0.944 0.994 2.059 0.287 0.362 0.267
x4 0.312 0.367 0.287 2.150 1.112 1.091
x5 0.344 0.385 0.362 1.112 2.117 1.041
x6 0.226 0.315 0.267 1.091 1.041 2.016
```

- Color-coded correlation matrix



3.1.2 Observed and latent variables

- Observed variables
  - 6 variables
  - **These are all Y variables**: they are predicted by the latent variables
- Latent variables
  - These are the X variables
  - They are the **factors**
  - We *create* them in the analysis

3.2 Output of the analysis

3.2.1 Data reduction

- The idea behind FA is to reduce the number of variables

  - Start with **6 items**
    - Want **fewer** than 6 factors
    - How many fewer?
- I simulated the data to have 2 "clumps"
  - We talked about this the past few weeks
  - So I'll show you a **2 factor model** *to start*

3.2.2 FA results

1. Loadings
   - Relation between latent factor (X) and observed variable (Y)
     - Matrix with rows = # items, columns = # **factors**
     - High loading = that X is highly related to that Y
   - Think: correlation or standardized regression coefficient
     - Range from -1 to 1

3.2.3 Model results: Loadings in R

```
Loadings:
   PA1    PA2
x1  0.535  0.422
x2  0.590  0.420
x3  0.549  0.446
x4  0.605 -0.417
x5  0.607 -0.368
x6  0.573 -0.424

                 PA1   PA2
SS loadings    1.999 1.043
Proportion Var 0.333 0.174
Cumulative Var 0.333 0.507
```

Factor Matrix[a]		
	Factor	
	1	2
x1	.535	.422
x2	.590	.420
x3	.549	.446
x4	.605	-.417
x5	.607	-.368
x6	.573	-.424

Extraction Method: Principal Axis Factoring.

a. 2 factors extracted. 7 iterations required.

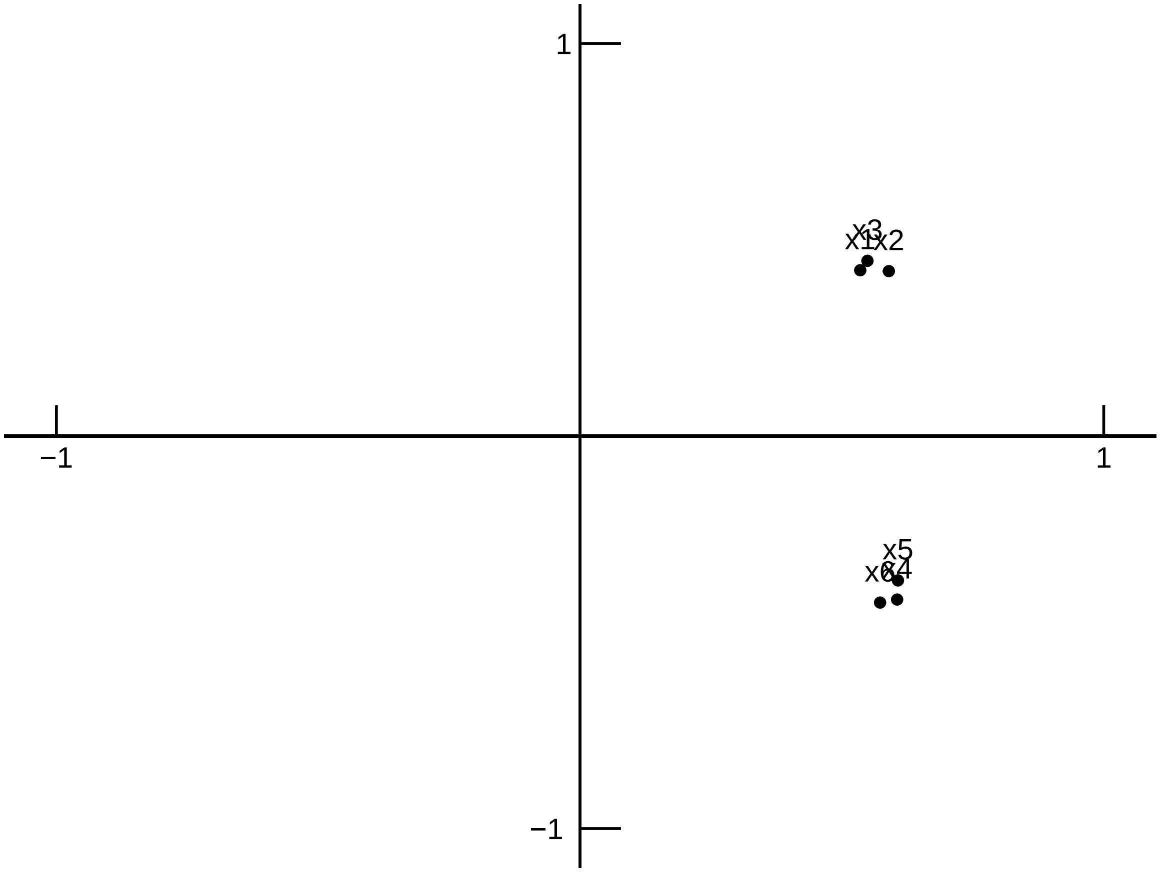
Total Variance Explained

Factor	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.490	41.506	41.506	1.999	33.309	33.309
2	1.537	25.612	67.118	1.043	17.383	50.691
3	.528	8.806	75.924			
4	.499	8.315	84.239			
5	.482	8.027	92.267			
6	.464	7.733	100.000			

Extraction Method: Principal Axis Factoring.

3.2.4 Model results: Loadings in SPSS

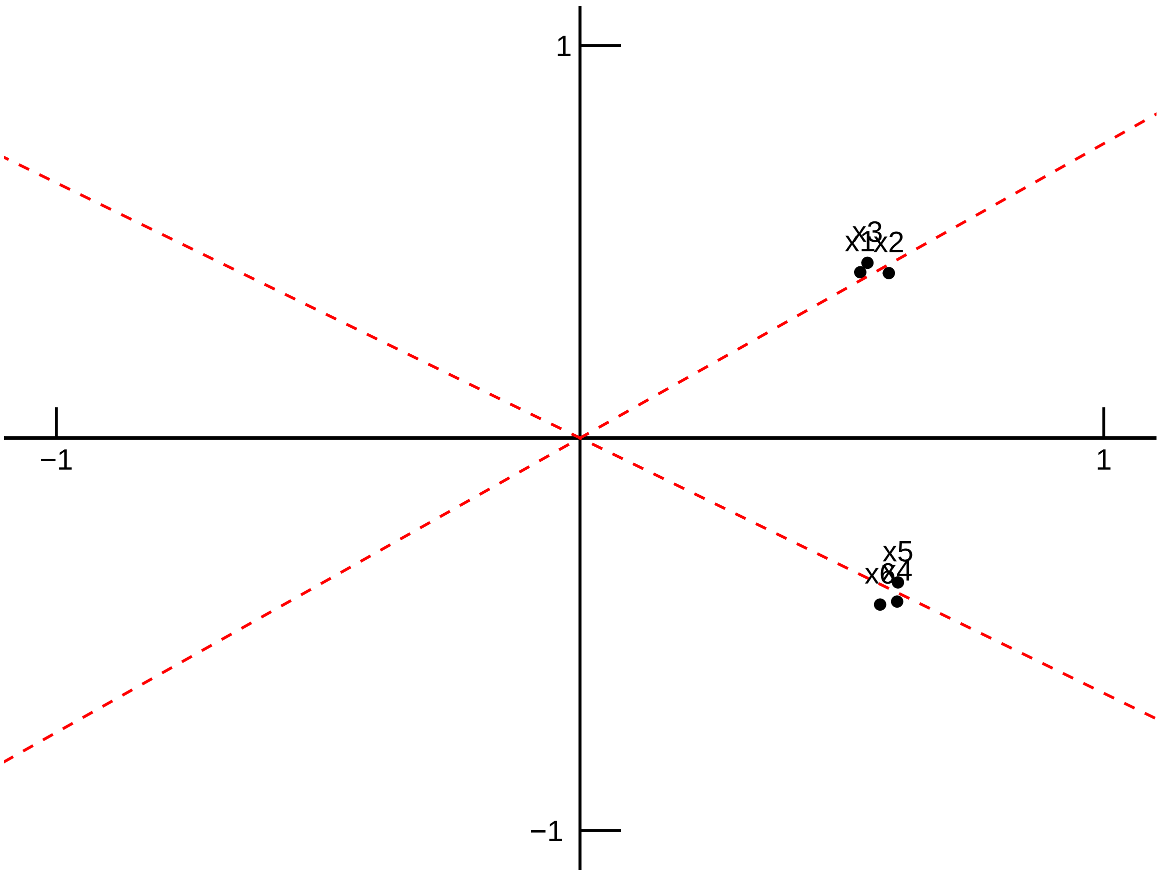
3.2.5 Loadings



3.2.6 Simple structure and rotation

- Solution has **simple structure** if each item has **high loadings** on only one factor and **near zero loadings** on all other factor
  - i.e., points are near the axes
  - Easier to interpret: items only relate to one axis
- **Rotated solution** rotates the axes to get closer to *simple structure*
  - We'll look at some different ways to rotate the solution
    - Conceptual version now
  - Easier to **interpret** a solution that has simple structure

3.2.7 Loadings on rotated axes



3.2.8 FA results

2. Communalities

    - Remember that we don't retain all the factors
    - Communalities are the proportion of variance in Y that's explained **by the factors (X) that you do retain**
    - Think: $R^2_{multiple}$ for Xs predicting Ys
        - This is the normal order (unlike PCA): X predicts Y

3.2.9 Model results: Communalities in R

```
       x1        x2        x3        x4        x5        x6 
0.4635790 0.5250197 0.5010631 0.5416307 0.5028439 0.5079958 
```

3.2.10 Model results: Communalities in SPSS

Communalities

	Initial	Extraction
x1	.318	.465
x2	.352	.524
x3	.334	.500
x4	.368	.540
x5	.355	.504
x6	.349	.508

Extraction Method: Principal Axis Factoring.

3.2.11 FA overview

- **Loadings** tell us *how items are correlated with factors*
  - Simple structure makes loadings more interpretable
  - Use rotation to try to get simple structure
- **Communalities** tell us how much *variance* in the items is *explained* by *the factors we kept*

4 FA details

4.1 Exploratory vs confirmatory

4.1.1 Exploratory vs confirmatory FA

- Two kinds of factor analysis

    - Obviously related, but also different models

1. **Exploratory factor analysis (EFA)** in this course
    - It is a "classic multivariate technique"
2. **Confirmatory factor analysis (CFA)** discussed briefly
    - CFA falls under structural equation modeling (SEM)

4.1.2 Exploratory factor analysis (EFA)

- EFA **explores** the factor structure of the variables
    - Largely *atheoretical*
    - *Discover* how many factors may be present
    - Few (if any) pre-conceptions about *which items may have high or low loadings on which factors*

4.1.3 Confirmatory factor analysis (CFA)

- CFA **confirms** a pre-existing factor structure
    - Requires *theory* to construct
    - Hypothesize a *specific number of factors*
    - Allow each item to *load on only one factor*
        - All other loadings are 0

4.1.4 EFA vs CFA: matrix of loadings

- EFA

Item	F1	F2	F3
1	0.618	0.094	-0.049
2	0.440	-0.075	0.065
3	0.671	0.037	0.041
4	0.031	0.731	-0.079
5	0.126	0.705	0.053
6	0.265	0.296	0.603
⋮	⋮	⋮	⋮

- CFA

Item	F1	F2	F3
1	0.620	0	0
2	0.450	0	0
3	0.665	0	0
4	0	0.725	0
5	0	0.689	0
6	0	0	0.613
⋮	⋮	⋮	⋮

- Zeroes are “fixed”: we specify that **those loadings are 0** so don’t estimate them

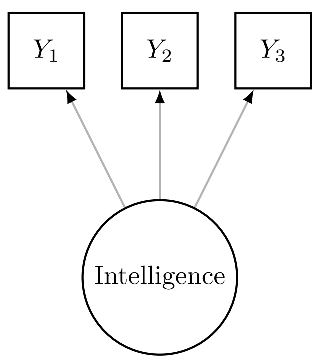
4.2 FA model

4.2.1 FA model

$$Y_i = T_i + e_{ij}$$

- FA partitions variance in each item into
  - **Common portion**: due to latent factors / true scores
  - **Unique portion**: due to error
- Big idea: Any correlations between items are due to what they have in common (i.e. a **common latent factor**)

4.2.2 FA model



4.2.3 FA model

- Partition variance of each item into **common** and **unique** portions
  - **Common portion**: due to latent factors
    - Correlations between variables due to common latent factor
  - **Unique portion**: Error (measurement or otherwise)

4.2.4 FA model

- There are p variables (items) and m factors
- $\mathbf{R}_{YY} = \mathbf{A}\mathbf{R}_F\mathbf{A}' + \mathbf{D}^2$
  - $\mathbf{R}_{YY} = p \times p$ matrix of **observed item correlations**
  - $\mathbf{A} = p \times m$ matrix of **loadings**
  - $\mathbf{R}_F = m \times m$ matrix of **correlations between factors**
  - $\mathbf{D}^2 = p \times p$ matrix of **unique variances**

4.2.5 Common variance portion

$\mathbf{A}\ \mathbf{R}_F\ \mathbf{A}' =$

$$\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \\ a_{4,1} & a_{4,2} \\ a_{5,1} & a_{5,2} \\ a_{6,1} & a_{6,2} \end{bmatrix} \begin{bmatrix} \sigma^2_{F_1} & \sigma_{F_1F_2} \\ \sigma_{F_2F_1} & \sigma^2_{F_2} \end{bmatrix} \begin{bmatrix} a_{1,1} & a_{2,1} & a_{3,1} & a_{4,1} & a_{5,1} & a_{6,1} \\ a_{1,2} & a_{2,2} & a_{3,2} & a_{4,2} & a_{5,2} & a_{6,2} \end{bmatrix}$$

- The common (shared) portion of the variance involves the **correlations among factors** ($\mathbf{R}_F$) and the **loadings** ($\mathbf{A}$)

4.2.6 Unique variance portion

- The matrix of "uniquenesses" is a **diagonal** matrix
  - Items are related only through **common factor**
  - There are *no correlations between uniquenesses across variables*

$$\mathbf{D} = \begin{bmatrix} d_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_6 \end{bmatrix}$$

- d_1 is the "uniqueness" for item 1
  - d_1^2 is the unique **variance** for item 1

4.3 Extraction methods

4.3.1 Extraction methods

- Two kinds of **extraction** (estimation) for EFA

1. **Principal axis factoring** (PAF), also called principal factor analysis
2. **Maximum likelihood factor analysis** (MLFA)

- Either method is fine, but use PAF if items are *not normally distributed*
  - Fabrigar, Wegener, MacCallum, & Strahan (1999)
  - Osborne & Costello (2005)

4.3.2 Principal axis factoring

- Uses "reduced" correlation matrix ($\mathbf{R}_{reduced}$)
  - Diagonal of 1s replaced with communalities
    - Why? More in a minute
- Perform PCA on reduced correlation matrix
- Iterate between loadings and correlation matrix until **observed correlations** and **model-implied correlations** are sufficiently close
  - What? More in a minute

4.3.3 Maximum likelihood factor analysis

- Uses a **weighted** version of $\mathbf{R}_{reduced}$
  - Diagonal elements are divided by that item's uniqueness (d_i)
    - Why? More in a minute
- Perform PCA on weighted reduced correlation matrix
- Iterate between loadings and correlation matrix until **observed correlations** and **model-implied correlations** are sufficiently close
  - What? More in a minute

4.3.4 Correlation matrix

$$\mathbf{R}_{YY} = \begin{bmatrix} 1 & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{21} & 1 & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & r_{32} & 1 & r_{34} & r_{35} & r_{36} \\ r_{41} & r_{42} & r_{43} & 1 & r_{45} & r_{46} \\ r_{51} & r_{52} & r_{53} & r_{54} & 1 & r_{56} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & 1 \end{bmatrix}$$

$$\mathbf{S}_{YY} = \begin{bmatrix} s_1^2 & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_2^2 & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & s_{32} & s_3^2 & s_{34} & s_{35} & s_{36} \\ s_{41} & s_{42} & s_{43} & s_4^2 & s_{45} & s_{46} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_5^2 & s_{56} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_6^2 \end{bmatrix}$$

- **Off-diagonal** elements are **common only**
  - Variables are related by what they have in **common**
- **Diagonal** involve **both common and unique**
  - FA only cares about common factors
  - Modify diagonal to make it **common only**

4.3.5 Principal axis factoring

$$\mathbf{R}_{reduced} = \begin{bmatrix} h_1^2 & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{21} & h_2^2 & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{31} & r_{32} & h_3^2 & r_{34} & r_{35} & r_{36} \\ r_{41} & r_{42} & r_{43} & h_4^2 & r_{45} & r_{46} \\ r_{51} & r_{52} & r_{53} & r_{54} & h_5^2 & r_{56} \\ r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & h_6^2 \end{bmatrix}$$

4.3.6 Principal axis factoring

- Elements on diagonal are **initial communalities**
  - What each variable has in **common** with all other variables
  - Squared multiple correlation (SMC) of that variable *predicted by all other variables*
    - h_1^2 is the $R^2_{multiple}$ for: $\hat{Y}_1 = b_0 + b_1 Y_2 + b_2 Y_3 + b_3 Y_4 + b_4 Y_5 + b_5 Y_6$

4.3.7 Principal axis factoring

- Remember PCA?
  - No measurement error
  - Initial communalities = 1.00
    - Using correlation matrix in PCA: 1s on diagonal
  - So actually the same idea
    - Diagonal is common variance only
    - In PCA, **everything** is common variance

4.3.8 Principal axis factoring

- Perform a PCA on the **reduced correlation matrix**
- Pattern of eigenvalues will be similar for PCA and PAF
  - Diagonal is reduced from all 1s to initial communalities
  - Eigenvalues are similarly reduced
  - Scree plot is the same shape, just shifted down for PAF

4.3.9 ML factor analysis

- Uses a **weighted** version of $\mathbf{R}_{reduced}$
  - Weights each initial communality by the inverse of its uniqueness
  - h_1^2 in PAF $\rightarrow \frac{h_1^2}{d_1^2}$ in MLFA
  - Increases values of main diagonal
  - Eigenvalues are larger compared to PAF or PCA
  - Scree plot can be different shape from PAF and PCA

4.3.10 Iterations in PAF and MLFA

- $\mathbf{R}_{reduced}$ to estimated loadings: $\hat{\mathbf{A}}_1$
  - Loadings to estimated correlation matrix: $\mathbf{R}_{estimated1} = \hat{\mathbf{A}}_1 \hat{\mathbf{A}}_1'$
- $\mathbf{R}_{estimated1}$ to estimated loadings: $\hat{\mathbf{A}}_2$
  - Loadings to estimated correlation matrix: $\mathbf{R}_{estimated2} = \hat{\mathbf{A}}_2 \hat{\mathbf{A}}_2'$
- $\mathbf{R}_{estimated2}$ to estimated loadings: $\hat{\mathbf{A}}_3$
  - Loadings to estimated correlation matrix: $\mathbf{R}_{estimated3} = \hat{\mathbf{A}}_3 \hat{\mathbf{A}}_3'$
- Repeat until the difference between estimated and observed correlation matrices is "small enough"

4.3.11 Heywood cases

- The iterative process sometimes causes problems
  - Heywood case = communality > 1 or loading > 1
- Causes: too few cases, bad start values, too many factors, too few factors, non-linear relationships between factors
- Some solutions:
  - Too few cases: drop items or add cases
  - Bad start values: use highest correlation of item with a **single other item** instead of SMC for initial communality

4.4 Summary

4.4.1 Summary

- EFA divides variance into
  - **Common** (latent factors)
  - **Unique** (error)
- Two approaches to estimating
  - Principal axis factoring (PAF)
  - Maximum likelihood (ML)

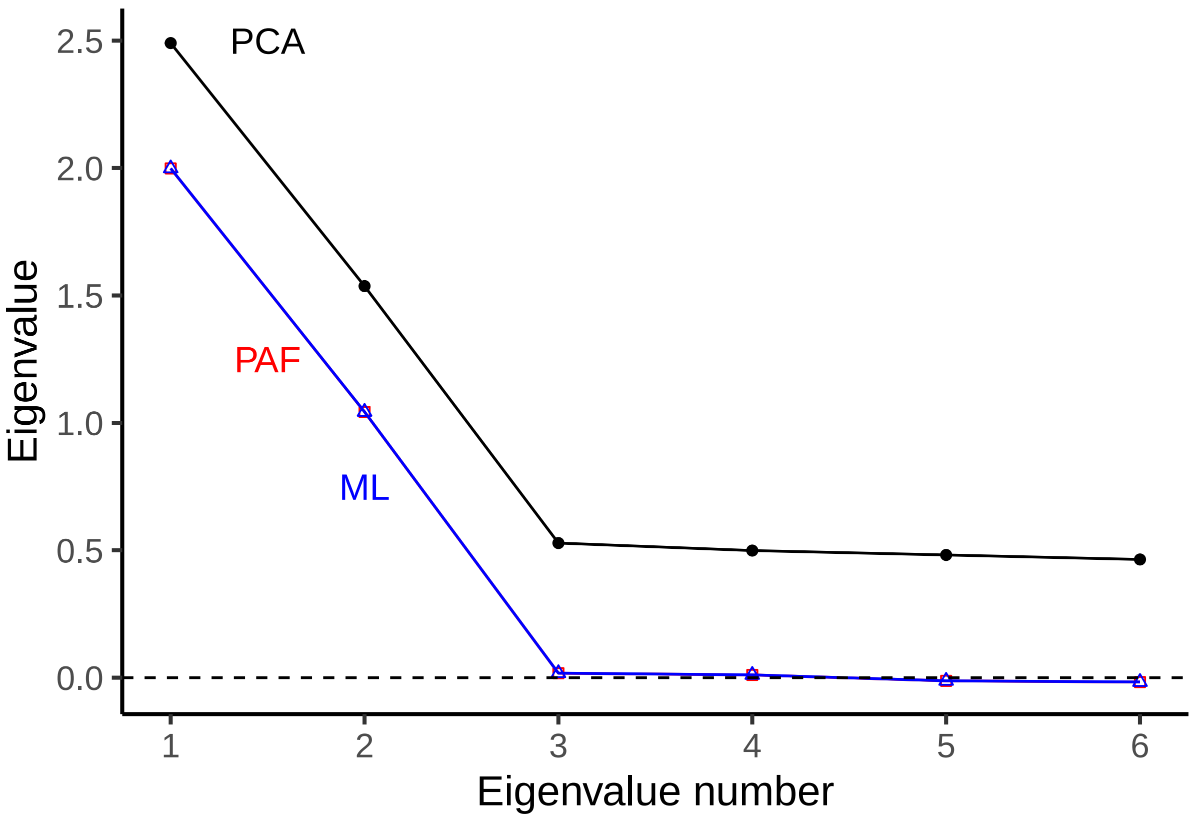
5 Number of factors and rotation

5.1 How many factors?

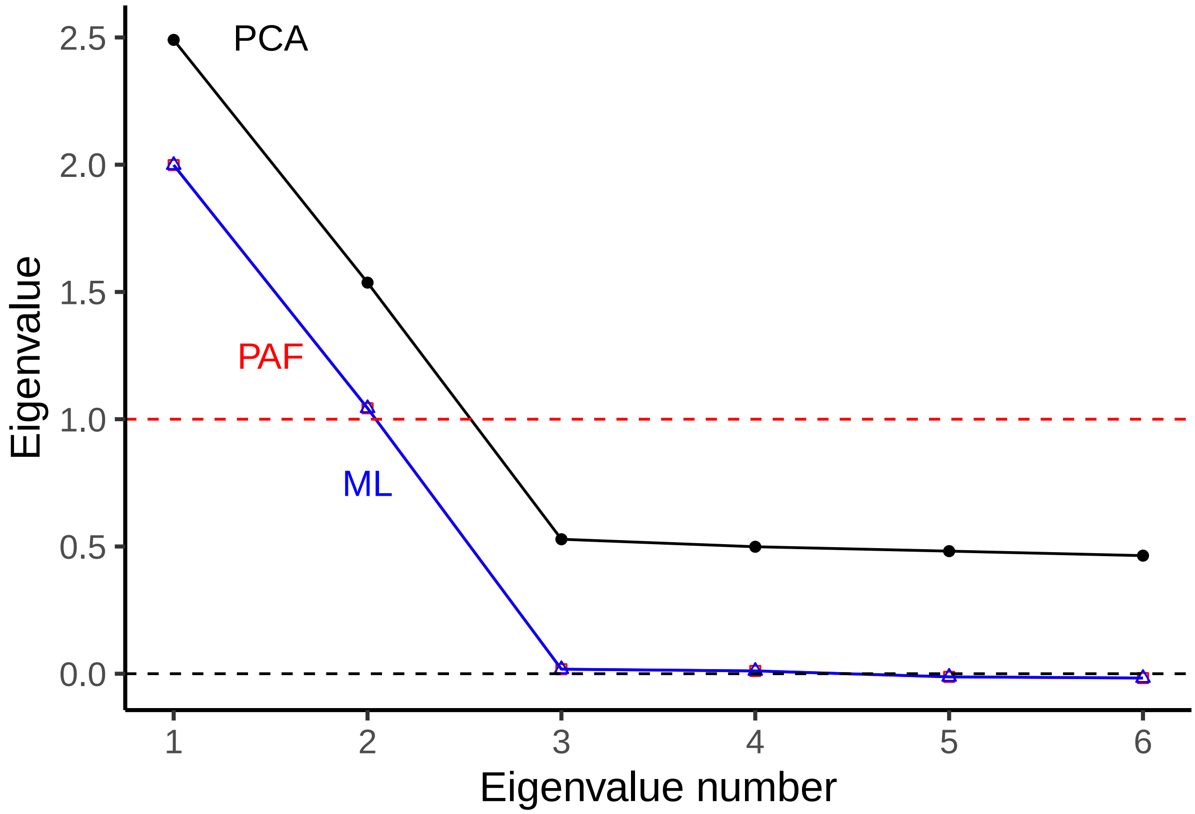
5.1.1 How many factors?

- Same options to pick number of factors as PCA
  - Bad: Kaiser criteria
  - Ok: Scree plot, proportion of variance accounted for
  - Good: Parallel analysis, MAP test
  - Also: Solution makes sense / theory
  - For MLFA: chi-square test

5.1.2 Scree plots

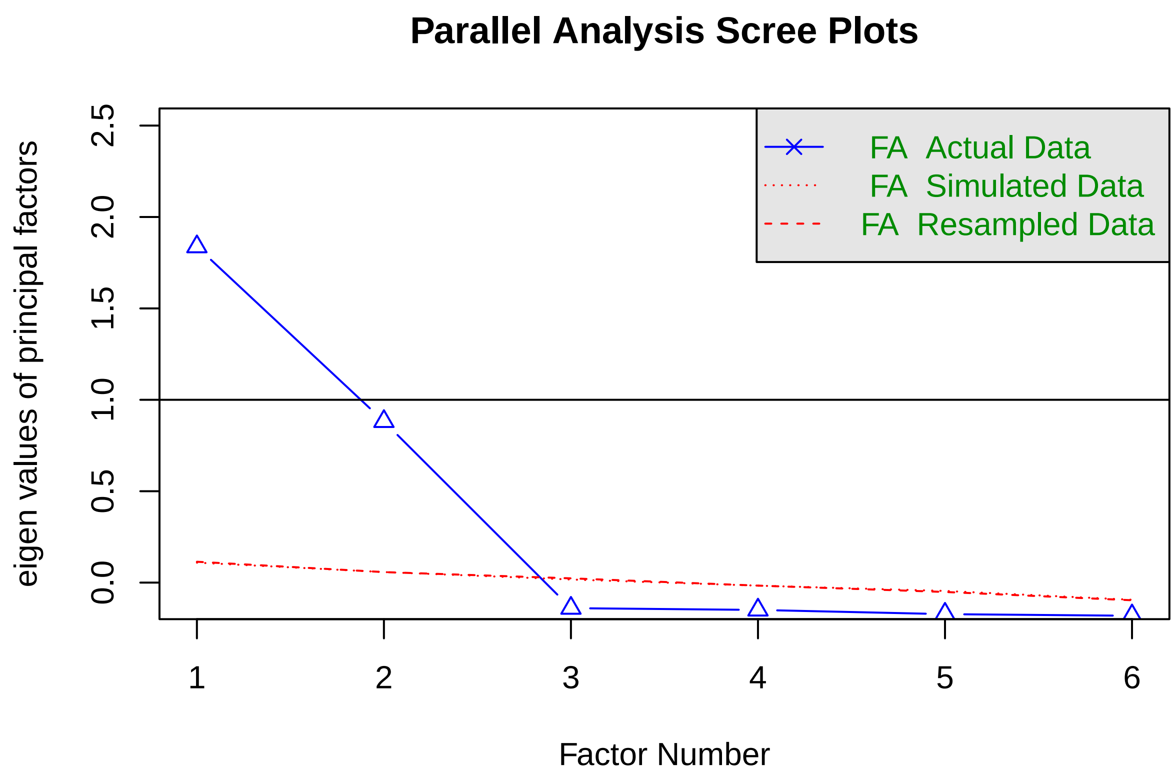


5.1.3 Kaiser criteria



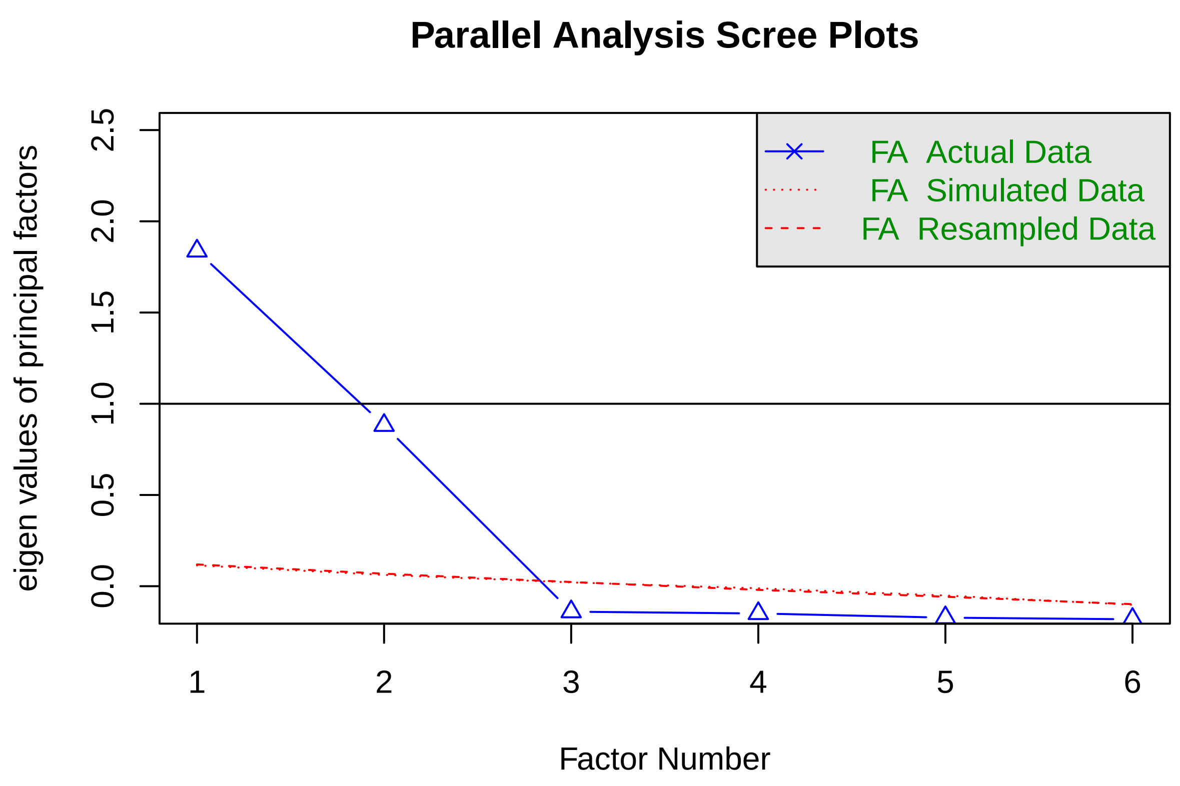
5.1.4 Parallel analysis in R

- PAF




```
Parallel analysis suggests that the number of factors =  2  and the number of components =  N
```

- MLFA




```
Parallel analysis suggests that the number of factors =  2  and the number of components =  N
```

5.1.5 Parallel analysis in SPSS

- SPSS gives you the eigenvalues for the original correlation matrix, not the reduced one, so...

5.1.6 MAP test in R

- PAF
  - Error: "imaginary eigen value"
  - No idea why
- MLFA

```
Run MATRIX procedure:

PARALLEL ANALYSIS:

Principal Axis / Common Factor Analysis

Specifications for this Run:
Ncases     1000
Nvars         6
Ndatsets   1000
Percent      95

Random Data Eigenvalues
         Root          Means       Prcntyle
     1.000000        .112143        .157845
     2.000000        .059266        .092894
     3.000000        .020263        .044896
     4.000000       -.014555        .005881
     5.000000       -.051602       -.026956
     6.000000       -.095223       -.064847
```

Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings		
Factor	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	2.490	41.506	41.506	1.999	33.309	33.309
2	1.537	25.612	67.118	1.043	17.383	50.691
3	.528	8.806	75.924			
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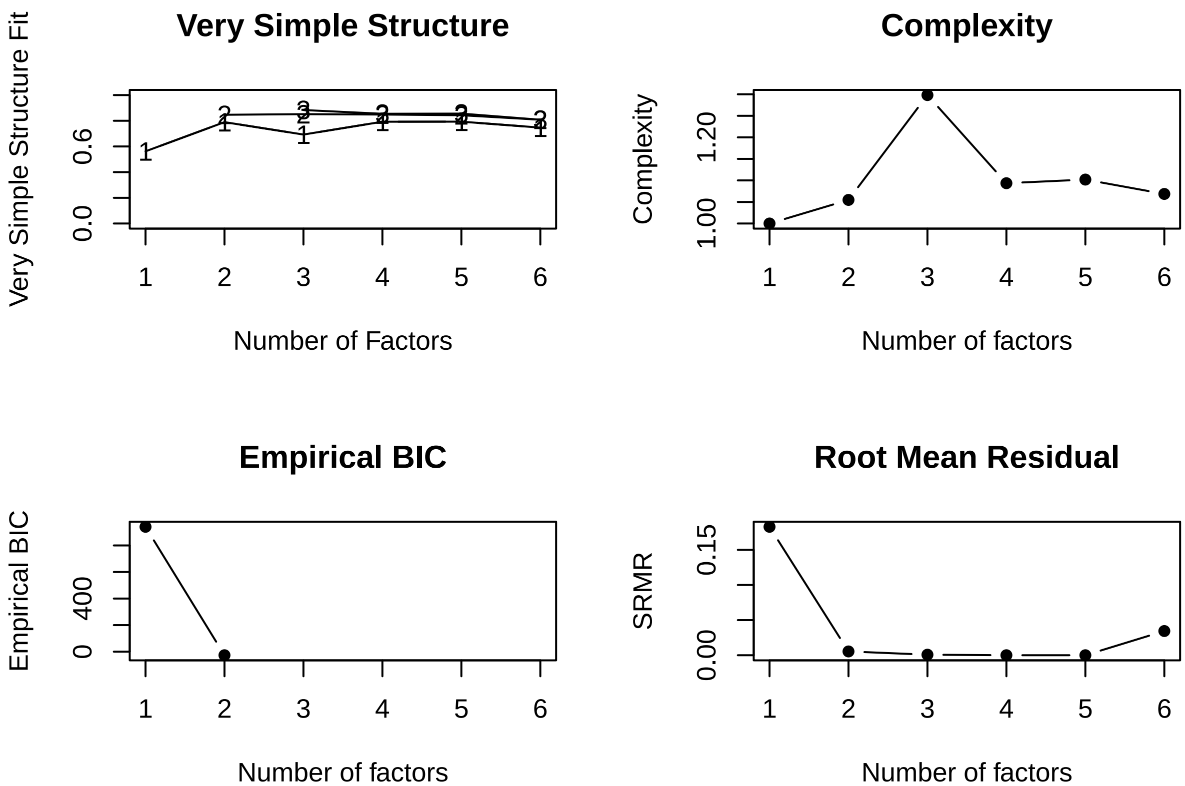
Extraction Method: Principal Axis Factoring.




```
Number of factors
Call: vss(x = x, n = n, rotate = rotate, diagonal = diagonal, fm = fm,
    n.obs = n.obs, plot = FALSE, title = title, use = use, cor = cor)
VSS complexity 1 achieves a maximimum of Although the vss.max shows  5  factors, it is probal
VSS complexity 2 achieves a maximimum of 0.85  with  3  factors
The Velicer MAP achieves a minimum of 0.1  with  2  factors
Empirical BIC achieves a minimum of  -26.76  with  2  factors
Sample Size adjusted BIC achieves a minimum of  -13.16  with  2  factors

Statistics by number of factors
  vss1 vss2  map dof   chisq     prob sqresid  fit RMSEA BIC SABIC complex
1 0.56 0.00 0.12   9 5.8e+02 3.1e-118     4.2 0.56  0.25 514   542     1.0
2 0.79 0.85 0.10   4 1.8e+00  7.8e-01     1.5 0.85  0.00 -26   -13     1.1
3 0.69 0.85 0.22   0 2.8e-02       NA     1.1 0.88    NA  NA    NA     1.3
4 0.79 0.85 0.42  -3 1.6e-09       NA     1.4 0.86    NA  NA    NA     1.1
5 0.79 0.84 1.00  -5 0.0e+00       NA     1.4 0.86    NA  NA    NA     1.1
6 0.75 0.81   NA  -6 2.6e+01       NA     1.8 0.81    NA  NA    NA     1.1
   eChisq    SRMR eCRMS eBIC
1 1.0e+03 1.8e-01  0.24  940
2 8.7e-01 5.4e-03  0.01  -27
```

```
3 1.7e-02 7.5e-04    NA   NA
4 8.5e-10 1.7e-07    NA   NA
5 2.7e-16 9.5e-11    NA   NA
6 3.6e+01 3.4e-02    NA   NA
```

5.1.7 MAP test in SPSS

```
Run MATRIX procedure:

Velicer's Minimum Average Partial (MAP) Test:

Eigenvalues
       2.4904
       1.5367
        .5284
        .4989
        .4816
        .4640

Average Partial Correlations
                       squared         power4
          .0000          .1180          .0268
         1.0000          .1246          .0223
         2.0000          .1003          .0250
         3.0000          .2156          .1155
         4.0000          .4226          .2868
         5.0000         1.0000         1.0000

The smallest average squared partial correlation is
        .1003

The smallest average 4rth power partial correlation is
        .0223

The Number of Components According to the Original (1976) MAP Test is
   2

The Number of Components According to the Revised (2000) MAP Test is
   1

------ END MATRIX -----
```

5.1.8 Chi-square test: ML only

- Null hypothesis: This number of factors is sufficient

- Alternative hypothesis: Need more factors
- R

"The total number of observations was 1000 with Likelihood Chi Square = 1.77 with prob < 0.78"

```
Factor Analysis using method =  ml
Call: fa(r = FA_data, nfactors = 2, rotate = "none", SMC = TRUE, warnings = TRUE,
    fm = "ml")
Standardized loadings (pattern matrix) based upon correlation matrix
    ML1   ML2   h2   u2 com
x1 0.52  0.44 0.46 0.54 1.9
x2 0.58  0.44 0.53 0.47 1.9
x3 0.53  0.46 0.50 0.50 2.0
x4 0.62 -0.40 0.54 0.46 1.7
x5 0.62 -0.35 0.50 0.50 1.6
x6 0.59 -0.41 0.51 0.49 1.8

                       ML1  ML2
SS loadings           2.00 1.04
Proportion Var        0.33 0.17
Cumulative Var        0.33 0.51
Proportion Explained  0.66 0.34
Cumulative Proportion 0.66 1.00

Mean item complexity =  1.8
Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  15  and the objective function was  1.49 with
The degrees of freedom for the model are 4  and the objective function was  0

The root mean square of the residuals (RMSR) is  0.01
The df corrected root mean square of the residuals is  0.01

The harmonic number of observations is  1000 with the empirical chi square  0.87  with prob <
The total number of observations was  1000  with Likelihood Chi Square =  1.77  with prob <

Tucker Lewis Index of factoring reliability =  1.006
RMSEA index =  0  and the 90 % confidence intervals are  0 0.032
BIC =  -25.86
Fit based upon off diagonal values = 1
Measures of factor score adequacy
```

```
                                              ML1  ML2
Correlation of (regression) scores with factors   0.90 0.82
Multiple R square of scores with factors          0.80 0.68
Minimum correlation of possible factor scores     0.61 0.36
```

- SPSS

Goodness-of-fit Test

Chi-Square	df	Sig.
1.769	4	.778

5.2 Rotation

5.2.1 Rotated solutions

- Same purpose for rotation
  - Make the solution more interpretable and clean
- Same options for rotation in EFA as in PCA
  - Orthogonal rotation: varimax
  - Oblique rotation: oblimin, promax

6 Conclusion

6.1 Summary of this week

6.1.1 Summary of this week

- Factor analysis (FA)
  - Reduce # of variables (from p variables to $< p$ **factors**)
  - Loadings relate items to **factors**
  - Communalities are how much variance in each item is explained by latent factors
  - Focus on **common** variance due to latent factor
    - Also measurement error