Multivariate: MANOVA and repeated measures ANOVA

Table of contents

1 Goals

1.1 Goals

1.1.1 Goals of this section

- **Multiple measures** of the same thing or related things as an **outcome**
  - Possibly over time
- Want the variables **separate**: Not PCA / FA

- In this section:
  - MANOVA (this week)
  - Repeated measures ANOVA (this week)
  - Mixed models (next week)
  - Mediation (2 weeks)

1.1.2 Goals of this lecture

- Multivariate Analysis of Variance (MANOVA)
  - Outcome is **multivariate**: Several outcome variables
- Repeated measures ANOVA (RM ANOVA)
  - **Univariate**: *Single* outcome variable, measured multiple times
  - **Multivariate**: *Multiple* outcome variables
- Punchline: MANOVA is almost never a good choice
  - But multivariate RM ANOVA is a decent approach

2 MANOVA

2.1 Univariate to multivariate

2.1.1 Extending ANOVA to multiple outcomes

- Frequently interested in more than 1 outcome at a time
  - Anxiety
    - Test anxiety, minor stressor anxiety, general anxiety
  - Children's school achievement
    - Reading ability, reasoning ability, math ability
  - Performance on a task
    - Speed and accuracy

2.1.2 Could do GLM on each outcome but…

- …you (often) shouldn't
  - **Inflated type I error** due to multiple tests on *correlated* outcomes
  - Sometimes only the **combination** of the outcomes shows an effect
  - Ignore **relations between DVs**

2.1.3 Structure of this section

- Review (univariate) between-subjects ANOVA
    - One outcome
- Extend to multivariate version
    - Multiple related outcomes

2.1.4 Univariate analysis of variance (ANOVA)

- Independent variables (IVs) are **categorical groups**
    - e.g., treatment and control
- Independent variables are called **factors**
    - Not to be confused with latent factors
- **Single** outcome variable (DV)
    - Continuous, normally distributed

2.1.5 ANOVA hypotheses are about the means

- One factor ANOVA
    - k levels of the independent variable
    - Null hypothesis: All k group means are equal
        - $H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$

2.1.6 ANOVA hypotheses are about the means

- Two factor ANOVA
    - k levels of one IV, m levels of other IV
    - 3 null hypotheses
        - Main effect 1: All k means across factor 1 are equal
        - Main effect 2: All m means across factor 2 are equal
        - Interaction: All cell means are equal

2.1.7 Partitioned variation

- Partition the variation in scores into:
  - between-subject portion (group differences, $SS_{between}$)
  - within-subject portion (error, SS_{within})
  - $SS_{total} = SS_{between} + SS_{within}$
- Calculate based on *observed scores*, *group means*, *grand mean*
  - X_{fi} = score for subject f in condition i
  - $\bar{T}_i$ = mean for scores in condition i
  - $\bar{G}$ = grand mean of all scores in the study

2.1.8 Partitioned variation

- Between group variation:

$$SS_{between} = n\Sigma(\bar{T}_i - \bar{G})^2 =$$
$$n[(\bar{T}_1 - \bar{G})^2 + (\bar{T}_2 - \bar{G})^2 + \cdots + (\bar{T}_k - \bar{G})^2]$$

- Within group variation:

$$SS_{within} = \Sigma(X_{fi} - \bar{T}_i)^2 =$$
$$(X_{1i} - \bar{T}_i)^2 + (X_{2i} - \bar{T}_i)^2 + \cdots + (X_{ni} - \bar{T}_i)^2$$

2.1.9 Testing the hypothesis

$$MS_{between} = \frac{SS_{between}}{k-1}$$

$$MS_{within} = \frac{SS_{within}}{k(n-1)}$$

$$F = \frac{MS_{between}}{MS_{within}}$$

- Compare observed F to critical $F(k-1, k(n-1))$
  - Significant test = at least one of the k groups is different from the other groups

2.2 MANOVA model

2.2.1 Multivariate analysis of variance (MANOVA)

- Independent variables are **categorical groups**
  - e.g., treatment and control
- Independent variables are called **factors**
  - Not to be confused with latent factors
- **Multiple** outcome variables
  - p outcome variables
  - Continuous, normally distributed

2.2.2 What does MANOVA do with all those outcomes?

- MANOVA creates a **linear combination** of the p outcome variables
  - Constructed to *separate* the k groups as much as possible
  - "Maximally discriminating linear combination"
- Look for group differences on the linear combination
- If you can't find differences on the **maximally discriminating linear combination** of all the DVs, then there really really aren't group differences on the DVs

2.2.3 MANOVA questions

- Do the groups differ at all?
  - On the maximally discriminating linear combination
- If yes, post hoc:
  - Which DVs have groups differences?
  - Which groups differ on those DVs?

2.2.4 Covariation matrix of outcomes P

- Covariation matrix of the p DVs: $p \times p$ matrix
  - Multivariate extension of SS_{total}
- Just like ANOVA: Partitions into **between** (**H**) and **within** (**E**)

$$\mathbf{P} = \begin{bmatrix} SS_1 & SP_{12} & \cdots & SP_{1p} \\ SP_{21} & SS_2 & \cdots & SP_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ SP_{p1} & SP_{p2} & \cdots & SS_p \end{bmatrix}$$

2.2.5 Hypothesis matrix H

- Multivariate extension of $SS_{between}$: $p \times p$ matrix
  - Diagonal: between-group **variation** of each DV
  - Off-diagonal: **covariation** between means for pairs of DVs

$$\mathbf{H} = \begin{bmatrix} SS_{H,1} & SP_{H,12} & \cdots & SP_{H,1p} \\ SP_{H,21} & SS_{H,2} & \cdots & SP_{H,2p} \\ \vdots & \vdots & \ddots & \vdots \\ SP_{H,p1} & SP_{H,p2} & \cdots & SS_{H,p} \end{bmatrix}$$

2.2.6 Aside: H matrix for two-factor MANOVA

- For a one-factor MANOVA, there is a single **H** matrix
- For a two-factor MANOVA, there is a single **H** matrix
  - BUT it can be further partitioned into 3 matrices reflecting:
    * Main effect 1
    * Main effect 2
    * Interaction effect

2.2.7 Error matrix E

- Multivariate extension of SS_{within}: $p \times p$ matrix
  - Diagonal: within-group **variation** of each DV, added across k grp
  - Off-diagonal: error **covariation**, added across k groups

- **No between-group information in this matrix**

$$\mathbf{E} = \begin{bmatrix} SS_{E,1} & SP_{E,12} & \cdots & SP_{E,1p} \\ SP_{E,21} & SS_{E,2} & \cdots & SP_{E,2p} \\ \vdots & \vdots & \ddots & \vdots \\ SP_{E,p1} & SP_{E,p2} & \cdots & SS_{E,p} \end{bmatrix}$$

2.2.8 Partitioned variation

- ANOVA
  - $SS_{total} = SS_{between} + SS_{within}$
- MANOVA
  - Total variation = between-group variation + within-group variation
  - One factor: $\mathbf{P} = \mathbf{H} + \mathbf{E}$
  - Two factor: $\mathbf{P} = \mathbf{H}_{factor1} + \mathbf{H}_{factor2} + \mathbf{H}_{factor1*factor2} + \mathbf{E}$

2.2.9 Multivariate hypothesis tests (omnibus)

- ANOVA
  - Divide SS by their degrees of freedom to produce MS (variances)
  - F-statistic is ratio of MSs (variances)
- MANOVA
  - Use matrix equivalent of variance: **Determinant**
    - Determinant is "generalized variance" for a matrix
  - Create analogues to F-statistics
  - Unfortunately, it's not straight-forward

2.2.10 Multivariate hypothesis tests

- Four commonly used multivariate tests
  - Different ratio of determinants or eigenvalues
- Wilks' lambda: within / total
- Pillai's trace: between / total
- Hotelling's trace: between / within
- Roy's largest characteristic root: between / total

2.2.11 Wilks' lambda

- $\Lambda = \frac{|\mathbf{E}|}{|\mathbf{H}+\mathbf{E}|} = \frac{|\mathbf{E}|}{|\mathbf{P}|}$
  - where $|\mathbf{E}|$ is the determinant of $\mathbf{E}$
- H_0: no between-group variation, so $\mathbf{H}$ is all zeroes and ratio is 1
  - As group differences increase, $\Lambda \to 0$
- Effect size = eta squared = $\eta^2 = 1 - \Lambda$
  - η^2 = variance accounted for by the best linear combination of DVs

2.2.12 Pillai's trace

- Pillai's trace = $trace\left[\mathbf{H}(\mathbf{H}+\mathbf{E})^{-1}\right]$
  - where the **trace** of a matrix is the **sum of the diagonal elements**
- Conceptually:
  - Matrix representing **proportion of variation that is between-group**
  - Sum of *eigenvalues* from that matrix

2.2.13 Hotelling's trace

- Hotelling's trace = $trace\left[\mathbf{H}(\mathbf{E})^{-1}\right]$
  - where the **trace** of a matrix is the **sum of the diagonal elements**
- Conceptually:
  - Matrix representing **ratio of between- to within-group variation**
  - Sum of *eigenvalues* from that matrix

2.2.14 Roy's largest characteristic root

- Roy's greatest characteristic root = first eigenvalue of $\mathbf{H}(\mathbf{H}+\mathbf{E})^{-1}$
- Conceptually:
  - Matrix representing **proportion of variation that is between-group**
  - *First eigenvalue* from that matrix

2.2.15 Summary of multivariate tests

Test	Matrix	Range (H_0 to H_A)	In words	Function
Wilks	E/T	1 to 0	Error proportion	Determinant
Pillai	H/T	0 to 1	Between proportion	Trace
Hotelling	H/E	0 to ∞	Between to within ratio	Trace
Roy	H/T	0 to 1	Between proportion	1st eigenvalue

These tests are similar, but they differ in terms of **power** and **robustness to violations** of assumptions

2.2.16 Assumptions of MANOVA

- GLM: Multivariate normality of outcomes, linearity, etc
- "Homogeneity of variance-covariance matrices"
  - Error matrix is same in all groups and **E** is average
  - Multivariate extension of homogeneity of variance assumption
- Box's M test to test this assumption
  - Significant test means that assumption is violated
  - Sensitive: use $p<.001$, ignore unless *n*s very different across groups

2.2.17 Which test should I use???

- One factor MANOVA with k = 2 groups: All tests are identical
- **Recommended**: Pillai's trace
  - Robust to assumptions, powerful when DVs not highly corr
- **Recommended**: Wilks' lambda
  - Good power, relatively robust when assumptions probably met
- **Maybe use**: Roy's greatest characteristic root
  - Powerful when DVs highly corr, not robust to assumptions
- **Not recommended**: Hotelling's trace
  - OK when sample size is very large

2.3 Summary and alternatives

2.3.1 MANOVA

- Extends ANOVA to multiple outcomes
  - Many omnibus test options
  - Many follow-up options
  - Maximally discriminating linear combination?
  - Missing data, ANOVA framework only, time
- Quantitude says MANOVA must die

2.3.2 MANOVA questions

- Do the groups differ at all (on max discriminating linear comb.)?
  - This is what Pillai's trace, etc are testing
- **If yes, post hoc**:
  - Which DVs have groups differences?
  - Which groups differ on those DVs?
  - Enders, C. K. (2003). Performing multivariate group comparisons following a statistically significant MANOVA. Measurement and Evaluation in Counseling and Development, 36, 40-56.

2.3.3 When to use MANOVA?

- DVs are **highly negatively correlated**
  - Time to complete a task and number of errors on task
- DVs are **all moderately correlated** in either direction
  - Around ± 0.6 correlation
  - Not really high enough to support a latent factor
  - Repeated measures

2.3.4 When not to use MANOVA?

- DVs are **not really correlated**
  - MANOVA is unnecessarily complicated and wasteful
  - You don't gain anything by analyzing them together

- DVs are all **highly positively correlated**
  - MANOVA is unnecessarily complicated and wasteful
  - The variables are all basically the same thing

2.3.5 Alternatives to MANOVA

- Repeated-measures DVs:
  - **Repeated measures ANOVA**
  - **Mixed / multilevel / hierarchical linear models**
  - Latent growth models
- Separate univariate ANOVAs: esp *uncorrelated* DVs
- SEM / path model with multiple DVs
- Latent factor: esp highly *correlated* DVs

3 Repeated measures ANOVA

3.1 Overview / review

3.1.1 Between-subjects ANOVA

- Different subjects in each condition or cell of the design
  - 2 dimensions: subjects and variables

subject	condition	outcome
1	1	3
2	1	4
3	1	3
4	2	5
5	2	3
6	2	3
7	3	1

subject	condition	outcome
8	3	2
9	3	4

3.1.2 Between-subjects ANOVA: Partitioning

- Partition the variation in scores into:
  - between-subject portion (group differences, $SS_{between}$)
  - within-subject portion (error, SS_{within})
  - $SS_{total} = SS_{between} + SS_{within}$

3.1.3 Repeated-measures

1. Measure the **same DV** over **time**
   - e.g., anxiety level at 1 wk intervals after starting medication
2. Measure the **same DV** in each of a **set of related conditions**
   - e.g., anxiety level after CBT, after medication, etc.

- Multiple outcome measures that are **related**
  - Measured on the same person (not independent)
  - MANOVA: **related** dependent variables

3.1.4 Repeated-measures ANOVA

- Subjects are repeatedly measured / same subject in all conditions
  - 3 dimensions: subjects, variables (Y_1), treatment or time (T)

subject	$Y1\_T1$	$Y1\_T2$	$Y1\_T3$	$Y1\_T4$
1	3	1	2	5
2	4	5	1	3
3	3	3	3	3
4	5	2	4	2
5	3	4	4	5
6	3	3	4	4
7	1	1	4	5
8	2	5	2	1
9	4	4	5	2

3.1.5 Two ways to do repeated-measures ANOVA

- Univariate:
  - Standard repeated measures ANOVA
  - Treats the outcome as **one variable** that is **measured repeatedly**
- Multivariate:
  - Treats the outcome as a **multivariate outcome**
    - Single outcome made up of **several (related) variables**
      - Sound familiar?

3.1.6 Univariate: n subjects, k repeated measures

- Single outcome variable Y
  - "Univariate"
- T (time or treatment) is a predictor
  - Specific levels: $1, 2, \dots, k$
- Also called "tall" or "stacked" data format
  - Used in mixed models (next week)

3.1.7 Univariate: n subjects, k repeated measures

subject	T	Y
1	1	Y_{11}
1	2	Y_{12}
1	$\vdots$	$\vdots$
1	k	Y_{1k}
2	1	Y_{21}
2	2	Y_{22}
2	$\vdots$	$\vdots$
2	k	Y_{2k}
$\vdots$	3	$\vdots$
n	1	Y_{n1}
n	2	Y_{n2}
n	$\vdots$	$\vdots$
n	k	Y_{nk}

3.1.8 Multivariate: n subjects, k repeated measures

- Several related outcome variables Y
  - "Multivariate"
- T (time or treatment) is not an explicit predictor
  - Treated like waves
- Also called "wide" data format
  - Used in MANOVA

3.1.9 Multivariate: n subjects, k repeated measures

subject	$Y1\_T1$	$Y1\_T2$	...	$Y1\_T4$
1	Y_{11}	Y_{12}	...	Y_{1k}
2	Y_{21}	Y_{22}	...	Y_{2k}
3	Y_{31}	Y_{32}	...	Y_{3k}
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	Y_{n1}	Y_{n2}	...	Y_{nk}

3.2 Univariate RM ANOVA

3.2.1 Univariate approach to repeated measures

- Partition variation in scores into:
  - Between-**subject** variation
  - Within-subject variation, which is further partitioned into:
    * Treatment (or time) effects for individuals
    * Residual or random error

3.2.2 Univariate approach to repeated measures

- Y_{ij} = score for person i at time or treatment j
- $\bar{T}_j$ = mean score for treatment or time j
  - Up to k treatments or times
- $\bar{P}_i$ = mean score for person i

- Up to n subjects

- $\bar{G}$ = grand mean of all scores

3.2.3 Between-subjects variation

- Individual subjects' variation around the **grand mean**

$$SS_{between\ subject} = k\sum_{i=1}^{n}(\overline{P}_i - \overline{G})^2$$

$$= k[(\overline{P}_1 - \overline{G})^2 + (\overline{P}_2 - \overline{G})^2 + \cdots + (\overline{P}_k - \overline{G})^2]$$

- Similar to between-**groups** variation in ANOVA, but no groups here
  - People are "groups"

3.2.4 Within-subjects variation

- Individual subjects' variation around **their mean**
- For person i:

$$SS_{within\ person\ i} = \sum_{j=1}^{k}(Y_{ij} - \overline{P}_i)^2$$

$$(Y_{i1} - \overline{P}_i)^2 + (Y_{i2} - \overline{P}_i)^2 + \cdots + (Y_{ik} - \overline{P}_i)^2$$

- Add up across all n subjects: $SS_{within\ subject} = \sum_{i=1}^{n}\sum_{j=1}^{k}(Y_{ij} - \overline{P}_i)^2$

3.2.5 Within-subjects variation

- Within-subjects variation = **time (or treatment)** + **residual**
- Time variation = timepoint mean variation around grand mean

$$SS_{time} = n\sum_{j=1}^{k}(\overline{T}_j - \overline{G})^2$$

$$= n[(\overline{T}_1 - \overline{G})^2 + (\overline{T}_2 - \overline{G})^2 + \cdots + (\overline{T}_k - \overline{G})^2]$$

3.2.6 Within-subjects variation

- Within-subjects variation = **time (or treatment)** + **residual**
- Residual variation = any remaining variation

$$SS_{residual} = SS_{time \times subject} = SS_{within\ subject} - SS_{time}$$

3.2.7 Full partitioning of variation

- Keep in mind: No groups here at all

$$SS_{total} = SS_{between\ subject} + SS_{time} + SS_{residual}$$

Source	SS	df	MS	F
Between	$SS_{between\ subject}$	$n-1$	$MS_{between\ subject}$	
Within	$SS_{within\ subject}$	$n(k-1)$	$MS_{within\ subject}$	
–Time	SS_{time}	$k-1$	MS_{time}	$\frac{MS_{time}}{MS_{residual}}$
–Residual	$SS_{residual}$	$(n-1)(k-1)$	$MS_{residual}$	

3.2.8 Mixed effects ANOVA

- Between-subjects + within-subjects = "mixed ANOVA"
  - Unfortunate: too easy to confuse with "mixed models"
    - Also have several other names: Next week
- You can have BOTH within and between subjects factors in ANOVA
  - e.g., group (between) and time (within)
- Also look at the interaction
  - Does time effect vary across groups? Or vice versa?

3.2.9 Assumptions of univariate RM ANOVA

- About the covariance matrix of the outcomes

$$\mathbf{S}_{YY} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\ & & \sigma_3^2 & \sigma_{34} \\ & & & \sigma_4^2 \end{bmatrix}$$

- σ_1^2 = variance of outcome at time 1 / treatment 1
- σ_{12} = covariance between outcome at time /treatment 1 and outcome at time / treatment 2

3.2.10 Compound symmetry and sphericity

- **Compound symmetry** of the covariance matrix of outcomes
  - Homogeneity of variances (i.e., variances are all the same):
    - $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma_4^2$
  - Homogeneity of covariances (i.e., covariances are all the same):
    - $\sigma_{12} = \sigma_{13} = \sigma_{14} = \sigma_{23} = \sigma_{24} = \sigma_{34}$
- Actual assumption: **Sphericity**
  - Compound symmetry holds for **differences** between pairs of scores
  - Slightly weaker assumption

3.2.11 Plausibility of sphericity assumption

- T_1 through T_k are different trials or conditions in a *single session*
  - Sphericity may be plausible
- T_1 through T_k are *different time points*
  - Say, 9th, 10th, 11th, and 12th grades
  - Probably expect T1 and T2 to be more alike that T1 and T4
  - Sphericity is probably not very plausible

3.2.12 Violations of Assumptions

- Even if sphericity is plausible, it still may be **violated**
  - Very small violations can **greatly increase type I error rate**
- How to deal with violation of sphericity?
  - Adjust for violations of sphericity to return alpha to .05
  - Use **multivariate test** of repeated measures (next section)

3.2.13 Adjusting for sphericity violations

- **Lower bound correction**: Most conservative
  - Ignore repeated measures, treat as between subjects
  - Use critical $F(1, \text{n} - 1)$
- **Greenhouse-Geisser**: Middle of the road
  - $\hat{\epsilon}$ ranges from $\frac{1}{k-1}$ (severe violation) to 1 (sphericity)
  - Multiply degrees of freedom by $\hat{\epsilon}$
- **Huynh-Feldt**: Least conservative, smallest adjustment
  - Multiply degrees of freedom by $\tilde{\epsilon}$ (function of $\hat{\epsilon}$)

3.3 Multivariate RM ANOVA

3.3.1 Mutlivariate approach to repeated measures

- Multivariate extension of *paired t-test*
- Basically a MANOVA on specific set of difference scores
  - Multivariate tests (i.e., Wilks' lambda) as in MANOVA

3.3.2 Vector of differences

- $k - 1$ differences between combinations of k repeated scores
  - Must be **linearly independent**
  - Most common: Differences between adjacent pairs of means
- For a single subject i:

$$\underline{Y}'_{id} = \begin{bmatrix} d_{i1} \\ d_{i2} \\ d_{i3} \\ \vdots \\ d_{i,k-1} \end{bmatrix} = \begin{bmatrix} Y_{i1} - Y_{i2} \\ Y_{i2} - Y_{i3} \\ Y_{i3} - Y_{i4} \\ \vdots \\ Y_{i,k-1} - Y_{ik} \end{bmatrix}$$

3.3.3 Matrix of difference scores

- $n \times (k-1)$ matrix of difference scores is matrix of outcomes
  - Rows are subjects, columns are difference scores
  - k repeated measures: $k-1$ difference scores

$$\mathbf{Y}_d = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1,k-1} \\ d_{21} & d_{22} & \dots & d_{2,k-1} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{n,k-1} \end{bmatrix}$$

3.3.4 Covariance matrix of differences

- $(k-1) \times (k-1)$ covariance matrix of differences
  - $s^2_{d_1}$ is the variance of the $(T1 - T2)$ scores across n subjects
  - $s_{d_1 d_2}$ is the covariance between $(T1 - T2)$ and $(T2 - T3)$
  - Unlike univariate test, **no assumptions about this matrix**
- For 4 timepoints, this is a 3×3 matrix:

$$\mathbf{S}_d = \begin{bmatrix} s^2_{d_1} & s_{d_1 d_2} & s_{d_1 d_3} \\ & s^2_{d_2} & s_{d_2 d_3} \\ & & s^2_{d_3} \end{bmatrix}$$

3.3.5 Null hypothesis

- H_0: $k-1$ vectors of **mean differences** are simultaneously
  - All equal to each other AND all equal to 0
- NS test = no differences over time
  - All mean differences are 0
  - No adjacent differences are different from one another
- Significant test = differences over time
  - Some of the mean differences are not 0
  - Some adjacent differences are different from one another

3.3.6 Multivariate hypothesis tests

- Perform a MANOVA on the difference score matrix
  - Multivariate hypothesis tests:
    * Wilks' lambda
    * Pillai's trace
    * Hotelling's trace
    * Roy's largest characteristic root

3.4 Summary and comparison

3.4.1 Summary

- Univariate RM ANOVA
  - Single, repeatedly measured outcome
  - Sphericity assumption
- Multivariate RM ANOVA
  - Multiple, related outcomes
  - No sphericity assumption

3.4.2 Comparison

- Univariate approach: $F(k-1,(n-1)(k-1))$
  - Assumptions about covariance matrix (sphericity)
    * But can adjust if assumptions not met
  - Missing data results in loss of entire subject
- Multivariate approach: $F(k-1,n-k+1)$
  - No assumptions about structure of covariance matrix
    * (except that $n \geq k$ so it is invertable)
  - Missing data results in loss of entire subject

3.4.3 Recommendations: univariate vs. multivariate

- Univariate is preferred with small sample sizes
  - Sphericity holds (rare): More powerful, simpler, correct α
  - ALWAYS use univariate (with correction) if $n < k$
- Multivariate is preferred with large sample sizes
  - If sphericity doesn't hold (common): correct α
  - Do not use unless $n \geq k$
    - With BS factors: n in each BS group needs to be $\geq k$

4 Summary

4.1 Summary

4.1.1 Summary of this week

- MANOVA is a way to analyze multiple outcomes in one model
  - Almost never a good choice
  - Limited utility for repeated measures
- RM ANOVA has univariate and multivariate versions
  - Univariate has some easily violated assumptions
  - Multivariate is good but still limited
  - Missing data, ANOVA framework only, time

4.1.2 Next few weeks

- RM ANOVA (both) have shortcomings
  - Best for short-term or single-session studies
  - Does not capture the TIME aspect of longitudinal data
  - Requires same # of repeated measures for each subject
  - Not informative about individual growth
  - Focus on **average** differences over time and group differences
  - ANOVA framework, so only categorical predictors
- Mixed models, latent growth models solve many of these issues